Spectral central limit theorem for additive functionals of isotropic and stationary Gaussian fields

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Let $B = (B_x)_{x\in\mathbb{R}^d}$ be a collection of $N(0,1)$ random variables forming a real-valued continuous stationary Gaussian field on $\mathbb{R}^d$, with $C(x-y) = \mathbb{E}[B_x B_y]$ its continuous covariance function. Define

$$Y_t = \int_{\|x\|\le t} \varphi(B_x)dx$$

where $\varphi\colon \mathbb{R} \to \mathbb{R}$ is a locally bounded function with $\mathbb{E}[\varphi(N)^2] < \infty$, $N \sim N(0,1)$. Since the pioneering works from the 80s by Breuer, Dobrushin, Major, Rosenblatt, Taqqu and others, central and non-central limit theorems for Y_t have never ceased to be refined, extended and applied to increasingly numerous and diverse situations, to such an extent that it has become a field of research in its own right.

For example, in stochastic geometry one is interested in obtaining limit theorems for the excursion sets at level $L \in \mathbb{R}$ of Gaussian random fields, corresponding to $\varphi = \mathbb{1}_{[L,\infty)}$.

The common belief, representing the intuition that most specialists in the subject have developed over the past four decades, is that as $t \to \infty$ the fluctuations of Y_t around its mean are, in general (i.e. except possibly in very special cases), Gaussian when B has short-memory and non Gaussian when B has long memory and the Hermite rank R of φ is different from 1.

Our goal is to show that this intuition can be wrong, and not only marginally or in critical cases. We will indeed highlight a variety of situations where Y_t admits Gaussian fluctuations in a long memory context.

To achieve this goal, we introduce a spectral central limit theorem, which extends the conclusion of the famous Breuer–Major theorem to situations where $C \notin L^R(\mathbb{R}^d)$.

Our main mathematical tools are the Malliavin–Stein method and Fourier analysis techniques.

Based on a joint work with Ivan Nourdin, University of Luxembourg.