Minkowski tensors of convex bodies: determination, reconstruction and rotational integral formulae

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The *Minkowski tensors* of a convex body K in n-dimensional Euclidean space are generalizations of the intrinsic volumes (volume, surface area, Euler–Poincaré characteristic, ...) of K, but have values in the algebra of symmetric tensors. Due to Alesker's characterization theorem they play a central role in modern convex geometry and have repeatedly been used in stochastic geometry and in stereological applications. Roughly speaking, they capture certain shape and position information of K, unavailable from its intrinsic volumes.

The purpose of this talk is to give an overview over recent results, which make this statement precise. We will discuss

- *uniqueness:* Is K determined by certain Minkowski tensors?
- *reconstruction:* How can K be obtained from Minkowski tensors in case of uniqueness?
- *stability:* Can the (Hausdorff-)distance of two convex bodies with certain coinciding Minkowski tensors be controlled?

To give an example, the *volume tensor* of rank $r \in \mathbb{N}_0$ of a set K with volume 1 can be identified with the vector of all moments of order r of the uniform distribution on K. Hence, K is uniquely determined by *all* volume tensors. Among all full-dimensional convex polytopes with at most v vertices, volume tensors up to rank $2v - n - 1$ are sufficient for this purpose and an elegant reconstruction algorithm can be formulated. We will also discuss a corresponding stability result.

The use of Minkowski tensors in stereological applications also requires integral geometric formulae. The last part of my talk is devoted to describe recent advances in this field.