An integral characterization for a class of random probability measures

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Based on a joint work with Bartłomiej Błaszczyszyn and Thomas Lehéricy. Let ζ be a random probability measure on a Polish space $\mathbb{X}$ which distribution is given by:

$$\mathbf{E}[f(\zeta)] = \mathbf{E}[L(\zeta)f((1-U)\zeta + U\delta_X)]$$

for some random $((0,1] \times \mathbb{X})$-valued (U, X) independent of ζ and a non-negative function L. Under a restrictive hypothesis on L, we will see that ζ can be decomposed in law as $\zeta \overset{d}{=} \sum_{n\geq 1} P_n \delta_{X_n}$, where $((P_n, X_n))_{n\geq 1}$ is a random sequence such that $(P_n)_{n\geq 1}$ is a random allocation model (or "stick breaking process") and $(X_n)_{n\geq 1}$ is independent. Moreover, the law of the joint sequence can be fully derived from L and the law of (U, X). Under the additional assumption that the law of X be diffuse, we will see that a stronger equation characterizes Pitman–Yor processes with a diffuse base measure. This latter is an extension of a characterization of Dirichlet processes given by G. Last in [1]. The arguments rely partly on classical stochastic geometry tools. In particular, we will give a Slivnyak-like theorem for Pitman–Yor processes with diffuse base measure interpreted as point processes on $(0,1] \times \mathbb{X}$.

References

[1] Last G., *An integral characterization of the Dirichlet process*, J. Theoret. Probab. **33** (2020), no. 2, 918–930.